Projectile Motion

2004

C7. A football is kicked from a point O on a horizontal plane, giving the ball an initial speed $V\,\mathrm{m\,s^{-1}}$ at an angle α to the horizontal. Assuming that gravity is the only force acting on the ball:

(*a*) Show that the maximum height, H metres, attained by the football is given by

$$H = \frac{V^2}{2g}\sin^2\alpha. \qquad \textbf{3}$$

(*b*) A second identical football is kicked from O with the same initial speed $V\,\mathrm{m\,s^{-1}}$ but at angle of projection 2α to the horizontal $(2\alpha < \frac{1}{2}\pi)$. The maximum height attained by this football is h metres.

(i) Show that

$$h = 4H\left(1 - \frac{2gH}{V^2}\right). \qquad \textbf{3}$$

[Note that $\sin 2\alpha = 2\sin\alpha\cos\alpha$.]

(ii) Given that the maximum height attained by the second football is three times that attained by the first, find the angles of projection of each of the two footballs. **4**

2005 SP

8. A particle is projected from the origin O, under constant gravity of magnitude g, with a velocity whose horizontal component has magnitude U, and whose upward vertical component has magnitude V.

(*a*) Show that, relative to horizontal and vertical axes Ox and Oy respectively, the path of the projectile is given by the equation

$$2U^2y = 2UVx - gx^2. \qquad \textbf{4}$$

(*b*) Given that the particle passes through the points (4, 7·2) and (10,15), find the angle of projection to the horizontal. **6**

2006

A4. A golfer strikes a golf ball from O across a horizontal section of ground, giving the ball an initial speed of $V\,\mathrm{m\,s^{-1}}$ at an angle α to the horizontal.

(*a*) Show that the range, R metres, of the golf ball is given by

$$R = \frac{V^2}{g}\sin 2\alpha. \qquad \textbf{4}$$

(*b*) The golfer intends the ball to land between two points A and B on the horizontal section such that $OA = L$ metres, $OB = 2L$ metres and OAB is a straight line.

Given that the angle of projection of the ball is $15°$, show that the initial speed must satisfy

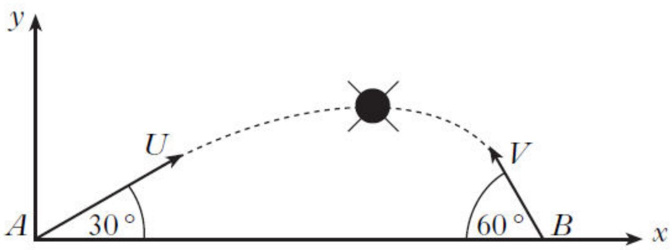
$$\sqrt{2} < \frac{V}{\sqrt{gL}} < 2. \qquad \textbf{3}$$

2012

A2. The greatest height reached by a projectile is one tenth of its range on horizontal ground. Calculate the angle of projection. **5**

2005

A10. Two points A and B are a distance L metres apart on horizontal ground. A ball is thrown from A towards B with speed U m s^{-1} at an angle of projection of 30 °. Simultaneously, a second ball is thrown from B towards A with speed V m s^{-1} and angle of projection 60 °.



(a) Using the coordinate system shown in the diagram:

(i) write down expressions in terms of U and t for the x and y coordinates of the ball thrown from A at time t seconds after projection; **2**

(ii) show that at time t, the x-coordinate of the ball thrown from B is $x = L - \frac{1}{2}Vt$ and write down the corresponding expression for the y-coordinate. **2**

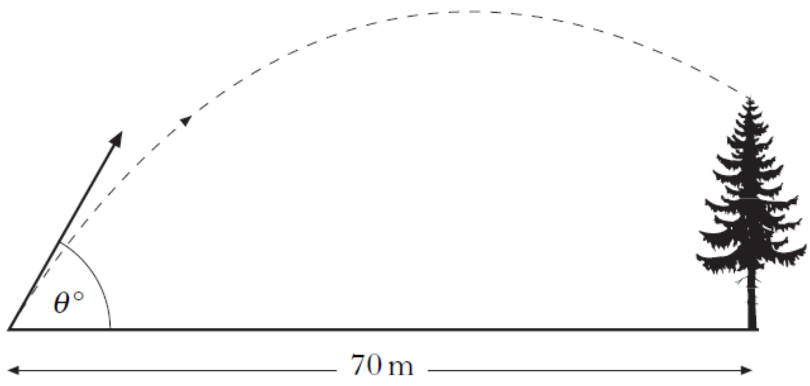
(b) The balls collide before reaching the ground.

(i) Show that $U = \sqrt{3}V$. **2**

(ii) Find an expression for the horizontal distance from A at which the collision takes place, giving your answer in terms of L. **4**

2007

A5. A golfer strikes a golf ball on a horizontal range, projecting the ball with speed 30 m s^{-1} at an angle $\theta°$ to the horizontal. After 3 seconds, the ball hits the top of a tree, which is situated at a horizontal distance of 70 metres from the point of projection.



Calculate the height of the tree. **4**

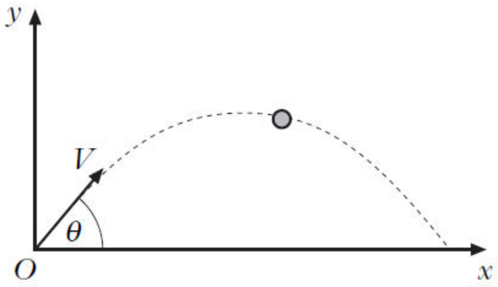
2009

A2. On a horizontal cricket field, a batsman strikes a cricket ball towards a fielder standing 40 metres away. The ball is projected from ground level at an angle $\theta°$ to the horizontal, where $\tan\theta° = \frac{3}{4}$, and is caught by the fielder when it is 2 metres above the ground, without having hit the ground first.

Calculate the speed with which the ball leaves the bat. **5**

2010

A9. Bobbie kicks a football from the origin O on a horizontal football pitch. The ball is projected at speed $V\,\mathrm{m\,s^{-1}}$ at an angle θ to the horizontal and moves freely under gravity.

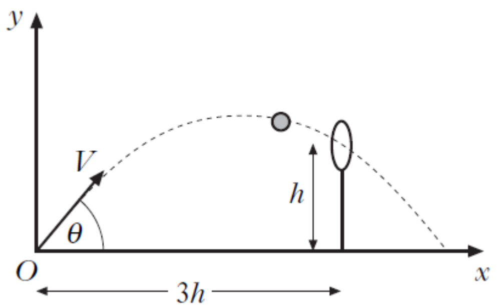


(*a*) Given that Ox and Oy are rectangular axes as indicated in the diagram, show from the equations of motion that the trajectory of the ball is given by

$$y = x\tan\theta - \frac{gx^2}{2V^2}(1+\tan^2\theta).$$ **3**

[Note that $\sec^2\theta = 1 + \tan^2\theta$.]

(*b*) The ball passes through the centre of a hoop with its trajectory unchanged. The centre of the hoop is at $(3h, h)$ and the speed of projection is given by $V = 3\sqrt{\frac{gh}{2}}$.



Determine the two possible values of $\tan\theta$. **4**

When $\tan\theta$ takes the larger of these values, find an expression for the range of the football in terms of h. **3**

2013

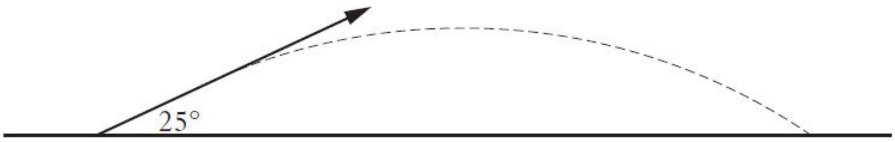
A10. Two projectiles are launched simultaneously from points A and B, where B is due East of A and situated on the same horizontal plane as A. The projectile launched from point A is projected towards B with speed $90\,\mathrm{m\,s^{-1}}$ at an angle of $30°$ to the horizontal. The projectile from point B is projected towards A with speed $50\,\mathrm{m\,s^{-1}}$ at an angle $\theta°$ to the horizontal.

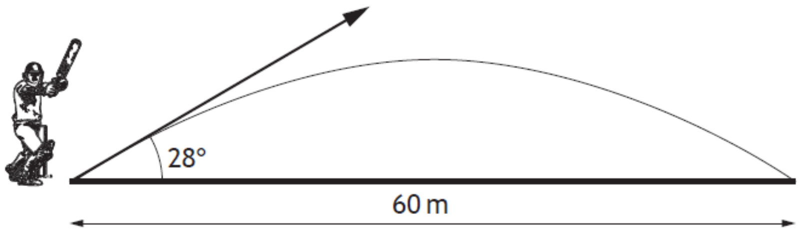
The two projectiles collide in mid-air at a distance d metres horizontally from point A.

Show that the height h at this point of collision is $h = \dfrac{d\left(4050\sqrt{3} - gd\right)}{12150}$. **4**

Find the angle of projection, $\theta°$, at which the projectile from B is launched. **4**

The projectiles collide 5 seconds after launch. Calculate the distance between A and B. **2**

2014	**A9.** An athlete competing in a long jump event accelerates from rest down the runway towards the take-off board. (*a*) On his first attempt, his acceleration can be modelled by the function $$a = 13\left(\frac{3}{8} - \frac{t}{16}\right)\text{m s}^{-2} \text{ for } 0 \le t \le \frac{5}{2}$$ where t is measured in seconds from the instant he sets off. (i) Find the speed reached when $t = \frac{5}{2}$. (ii) The athlete maintains this speed to the take-off board. Using this as the speed of projection, calculate the horizontal distance that the athlete would jump if his take-off angle is 25° to the horizontal, as shown below.  (*b*) On his second attempt, the athlete jumps a distance of 7·51 metres, having reached a take-off speed of $10{\cdot}2\text{ m s}^{-1}$. (i) Calculate the possible angles of projection for this jump. (ii) By considering the maximum height achieved in each case, show clearly which of these angles of projection represents his take-off angle.	 3 3 3 2
2016 SP	**15.** A golfer hits a ball from the point O with velocity $(P\mathbf{i} + Q\mathbf{j})\text{ m s}^{-1}$. The ball first hits the ground a distance of 50 metres from O in the horizontal plane. (a) Show that $PQ = 25g$. (b) Given that the ball passes through $45\mathbf{i} + 1{\cdot}6\mathbf{j}$ (i) Calculate P. (ii) Calculate the initial angle of projection to the horizontal.	 4 4 2
2016	**16.** A ball is projected from an origin on horizontal ground with speed $V\text{ m s}^{-1}$ at an angle of elevation of θ and moves freely under gravity. It passes through a point which is x metres horizontally from the origin at a height y metres above the ground. (a) Show that the trajectory of the particle has equation $$y = x\tan\theta - \frac{gx^2}{2V^2}\left(1 + \tan^2\theta\right).$$ (Note that $\sec^2\theta = 1 + \tan^2\theta$) (b) The ball is at a vertical height of h metres when it has travelled $4h$ metres horizontally. It is again at a height of h metres when it has travelled a further h metres horizontally. Determine the angle of projection θ.	 3 5

2017	**7.** A cricket batsman hits a ball from ground level. The ball lands on the boundary which is 60 metres away.  If the angle of flight to the horizontal ground is 28° at the instant the ball leaves the bat, calculate the initial speed of the ball.	**5**